Optimization of Cross-Hedging Strategies for Diverse Equity Portfolios: Identifying the Quintessential Asset Type

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Abstract

The increasing complexity of modern financial markets necessitates effective **risk management** and **portfolio yield optimization** strategies. This study investigates the efficacy of **cross-hedging** in **reducing risk** and improving performance within **diverse equity portfolios**. We hypothesize that cryptocurrency serves as an ideal cross-hedging instrument for a multi-sector, multi-asset equity portfolio. The decentralized nature, 24/7 trading, and potential for high returns of cryptocurrencies could offer unique hedging properties, possibly minimizing risk coefficients while maximizing returns. However, on the whole, operational challenges in managing multiple correlated positions simultaneously may increase transaction costs, potentially limiting strategy execution. Our methodology includes correlation analysis using Pearson's Product Moment Correlation Coefficient to identify suitable cross-hedging assets, determine optimal hedge ratios, and compute long and short positions. We evaluate daily returns, compare hedged and unhedged performance, and assess portfolio stability through various performance metrics and stress testing. Mean-variance optimization supports portfolio allocation, while Monte Carlo simulation strengthens volatility analysis by generating real-life probabilistic scenarios. The study's background lies in the evolution and advent of modern financial markets since the late 20th century, which has highlighted the need for advanced risk management techniques. Our rationale stems from the potential of cross-hedging to provide a balanced approach to risk mitigation and return optimization. The aim is to empirically identify the most effective cross-hedging asset for diverse equity portfolios. Our results will provide insights into the effectiveness of various assets as cross-hedging instruments, including their impact on risk reduction and return enhancement. Meanwhile, the implications of this research extend to both individual and institutional investors, offering a framework for implementing cross-hedging strategies to navigate market fluctuations effectively. This study contributes to the broader understanding of portfolio optimization techniques in contemporary financial markets.

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Introduction

What is cross-hedging?

Cross-hedging has roots dating back to ancient times when merchants engaged in trade across distant markets. However, its formalization as a risk management strategy gained momentum in the early 20th century with the development of modern financial markets. The concept evolved as investors sought ways to hedge against specific risks using financial instruments not directly related to their portfolios.

Importance of risk management

The importance of risk management has been emphasized throughout history. However, with the increasing complexity and globalization of financial markets, investors recognized the need for innovative risk management solutions. Cross-hedging became integral to this pursuit, allowing investors to navigate market uncertainties while preserving capital, decreasing risk, and optimizing returns.

Why is better than other strategies?

The evolution of cross-hedging gaining traction coincided with paralleling advancements in financial theory and technology. Traditional hedging strategies, prevalent until then, primarily relied on using identical or highly correlated assets for risk mitigation. Cross-hedging emerged as a more versatile strategy during this period, offering investors greater flexibility and effectiveness in managing risks across diverse asset classes and markets.

Magnitude

Cross-hedging's historical relevance lies in its ability to mitigate the magnitude of potential losses during periods of market turmoil. Throughout the 20th and 21st centuries, financial markets witnessed numerous crises, from the Great Depression to the Global Financial Crisis. Still, this strategy, with its emphasis on diversification and risk mitigation, has played a crucial role in minimizing the impact of such events on investment portfolios, underscoring its enduring importance in the realm of risk management.

Literature Review

Name of Study	Overarching Aim	Learnings for this Research	Limitations
Markowitz's "Portfolio Selection" (1952)	To introduce portfolio optimization and the mean-variance framework	Serves as a foundational work in portfolio optimization, emphasizing diversification benefits; provides insights into risk-return trade-offs and the importance of asset allocation	Limited applicability to cross-hedging strategies, as it does not directly address optimization for a diverse equity portfolio; may not consider specific asset correlations
Fama and French's "Common Risk Factors in the Returns on Stocks and Bonds" (1993)	To examine relationships between stocks and bonds, identify common risk factors affecting both asset classes	Contributes to understanding asset correlations and risk factors affecting portfolio returns; highlights the importance of diversification across asset classes	The study primarily examines common risk factors affecting stocks and bonds individually, potentially overlooking interactions between different asset classes within a diversified portfolio.
DeRoon, F. A., Nijman, T. E., & Werker, B. J. (2003). Testing for mean-variance spanning with short sales constraints and transaction costs: The case of emerging markets. Journal of Finance, 58(2), 713-733.	To investigate mean-variance spanning with short sales constraints and transaction costs	Offers insights into the mean-variance spanning property in the presence of short sales constraints and transaction costs; provides implications for portfolio optimization and risk management strategies	The study's examination of mean-variance spanning with short sales constraints and transaction costs is primarily focused on emerging markets, potentially overlooking nuances specific to developed markets. Differences in market structures, regulations, and investor behaviors between emerging and developed markets may limit the generalizability of findings to broader investment contexts, particularly in well-established financial markets.

Rationale

1. **Addressing Underexplored Areas:** This research seeks to fill the gap in existing literature by investigating cross-hedging strategies for diverse equity portfolios in niche markets or emerging asset classes where research is scarce, providing valuable insights into previously unexplored areas of risk management and portfolio optimization.

2. **Risk Factor Examination:** This research focuses on examining specific risk factors that traditional hedging strategies may overlook through a lens bounded by probability of volatility.

Hypothesis

Null Hypothesis (H0)

There is no significant difference in the risk-adjusted returns among various potential assets used for cross-hedging in a diverse equity portfolio.

Alternative Hypothesis (H1)

Certain potential assets used for cross-hedging demonstrate superior risk-adjusted returns compared to others in a diverse equity portfolio.

Key Words

Correlation

Correlation is a statistical measure that quantifies the degree to which two or more variables, in our case, financial assets, move in relation to one another. It provides insights into how closely or inversely related assets are. For example, if two stocks have a high positive correlation, they tend to move in the same direction, while a negative correlation suggests they move in opposite directions. Correlation is a critical concept in cross-hedging, as it helps identify assets that can effectively hedge each other due to their correlation behavior.

Adjusted Close Prices

Adjusted close prices are fundamental in financial analysis as they account for corporate actions like dividends, stock splits, and other adjustments that can impact the historical performance of an asset. Using adjusted close prices rather than normal close prices ensures the accuracy of historical performance assessment. For instance, if an investor were to assess the historical performance of a dividend-paying stock, using adjusted close prices would factor in the dividend payments and provide a more accurate representation of returns over time.

Hedge Ratio

The hedge ratio is a critical concept in hedging strategies. It refers to the ratio used to determine the appropriate allocation of a hedging asset within a portfolio. For example, if an investor aims to hedge a specific amount of risk in their equity portfolio, the hedge ratio specifies how much of a hedging asset should be held to offset that risk effectively.

Sharpe Ratio

The sharpe ratio is a widely used measure of the risk-adjusted return of a portfolio or investment. It helps in assessing the performance of a portfolio relative to the level of risk undertaken. For instance, if Portfolio A offers a higher return for the same level of risk as Portfolio B, it would have a higher Sharpe Ratio, making it a more attractive investment.

Dataset

Ticker	AGG	BTC-USD	CL	ES	ETH-USD	FXB	FXE	GC	GLD	LQD	...	REET	RWX	SLV	SMH	SPY	TLT	USO	UUP	VNQ	VTI
Date																					
2018-02-22	0.000094	-0.066261	0.003610	0.002090	-0.044662	0.001994	0.002617	0.642032	0.005080	0.001372	...	0.008319	0.002789	0.007692	-0.003381	0.001295	0.002994	0.024117	-0.003406	0.011317	0.000649
2018-02-23	0.002536	0.029166	0.013029	0.034035	0.061251	0.001327	-0.002363	0.000000	-0.001268	0.004618	...	0.014395	0.015076	-0.004480	0.022202	0.015814	0.008843	0.014196	0.002130	0.015871	0.015381
2018-02-24	0.000000	-0.048535	0.000000	0.000000	-0.027777	0.000000	0.000000	0.000000	0.000000	0.000000	...	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2018-02-25	0.000000	-0.015232	0.000000	0.000000	0.005097	0.000000	0.000000	0.000000	0.000000	0.000000	...	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
2018-02-26	0.000750	0.070116	0.010894	-0.010990	0.028594	-0.000147	0.001267	0.000000	0.002455	0.001279	...	0.004075	0.010666	0.005756	0.019959	0.011545	0.000338	0.007023	-0.001277	0.002798	0.010452

Cryptocurrency | Real Estate | Equity Securities | Currency Indexes | Futures | Sovereign and Corporate Bonds | Commodities

- 7 Asset Classes
- 3 Tickers for Each Class

- **Source:** Yahoo Finance (Historical Data)
- **Timespan:** (22-02-2018 to 14-02-2024)

Methodology - Data Collection and Preprocessing

The dataset we compiled encompasses a wide array of individual assets and commodities, selected based on their significance in the global financial landscape and their potential roles as hedging assets. Accordingly, we included a diversified portfolio of equities of various sectors and industries which acts as a benchmark for us to test our assets against.

In data preprocessing, we **transformed daily price data into logarithmic returns** to accurately capture portfolio movements. This step **accounts for compounding effects over time and mitigates the impact of market volatility** on our analysis. To achieve this, we **applied the natural logarithm function to the ratio of current and previous prices for each asset. Additionally, we added a constant of 1 to the returns ratio** to ensure valid calculations, **especially when dealing with zero or negative returns**. This logarithmic transformation makes returns more interpretable and allows for additive representation. This is **particularly advantageous** when working with **time series data** or when **comparing the performance of multiple assets.**

$$ln\left(\frac{P_t}{P_0}\right) = \ln(1+r) = lne^R$$

$$ln\left(\frac{P_t}{P_0}\right) = \ln(1+r) = Rlne$$

Log Return Formula, Where:

R = continuously compounded rate over the period

P_o = initial closing price of the stock

P_t = final closing price of the stock

r = simple return of the stock over time

Methodology - Correlation Analysis and Hedge Ratios

i. Correlation Analysis: -

1) **Correlation analysis** uncovers **interactions between equity portfolio and cross-hedging assets**.
2) **Pearson's product moment correlation coefficients (PMCC)** quantify pairwise correlations.
3) **PMCC assesses extent and direction of linear associations** between portfolio and assets.
4) Correlations were **measured using historical returns** to **assess performance** as well as the **magnitude and direction of movement** relative to the portfolio.
5) **Low or negative correlations** indicate assets' **potential for risk diversification.**
6) However, **correlations may vary over time** and may not capture all asset interactions.

ii. Hedge Ratio Computation: -

1) **This ratio** is essential for portfolio risk management, **determining optimal allocation of cross-hedging assets**.
2) **Calculated through linear regression** modeling **relationship between asset returns and portfolio**.
3) Regression model **fitted to historical return data** to estimate coefficients.
4) Slope of regression line represents hedge ratio, guiding **proportional exposure of each asset**.
5) Precise calibration of asset allocations **enhances portfolio resilience** against market uncertainties.

Methodology - Position Calculation and Return Analysis

iii. Long and Short Positioning: -

1) **Long and short positions** computed based on **previously calculated hedge ratios.**
2) **Long positions established for assets expected to increase in value,** allowing for potential capital appreciation.
3) **Short positions serve as hedges against potential losses,** offsetting downside risks within the portfolio.
4) **Long position is determined by multiplying the asset's hedge ratio by the total portfolio value. Short position is determined by multiplying the asset's negative hedge ratio by the total portfolio value.**
5) Allocation to long or short positions are guided by computed hedge ratios: **positive ratios for long positions**, **negative ratios for short positions.**

iv. Computing the daily returns for both long and short positions: -

After determining hedge ratios and establishing positions, we **calculate daily returns**. For **assets in long positions**, returns are **computed by multiplying the asset's daily price change by its hedge ratio**. Conversely, for **assets in short positions**, returns are **computed by multiplying the negative of the daily price change by its hedge ratio**.

v. Computing cumulative returns: -

After computing daily returns, we calculate **cumulative returns** by **sequentially adding the daily returns over the specified period**. This process **reflects the total growth or decline** in the portfolio value **over time**, providing a detailed measure of investment performance.

Methodology - Performance Metrics and Optimization

vi. Calculating Performance Metrics: -

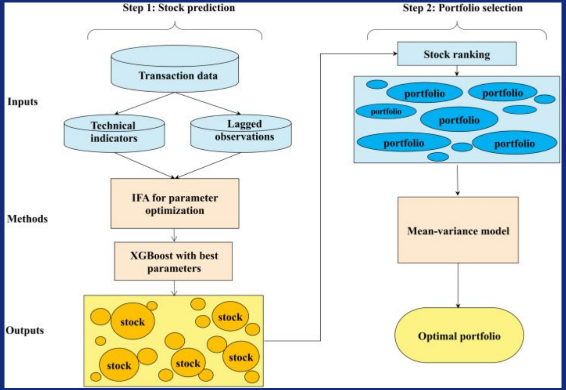
We **calculate performance metrics** for **each asset type**, **both hedged and unhedged**, to evaluate cross-hedging effectiveness. Metrics include **cumulative return, annualized return, annualized standard deviation, Sharpe ratio, and max drawdown.**

vii. Mean Portfolio Optimization (MPT) and Weight Allocation: -

1) **Mean-Variance Optimization (MPT)** constructs an **optimal portfolio** by balancing expected returns and portfolio risk.
2) **Allocation of weights to each equity** is determined to achieve an optimal risk-return profile.
3) MPT identifies the **most efficient asset combination that maximizes returns for a given risk level** or minimizes risk for a desired return level.
4) Utilizing the **covariance matrix of asset returns** and investor's risk preferences, MPT determines the **ideal asset allocation to optimize portfolio performance.**
5) **Monte Carlo Simulation plots an efficient frontier**, aiding in visualizing optimal portfolio choices.

vii. Monte Carlo: -

Monte Carlo simulation is employed to generate **thousands of possible portfolio scenarios**, allowing for a realistic analysis of possible outcomes. By **randomly sampling** from the **historical returns distribution of each asset**, we create a range of possible portfolio returns and risk levels.



Pipeline for portfolio optimization through weight allocation using the mean-variance model

Results

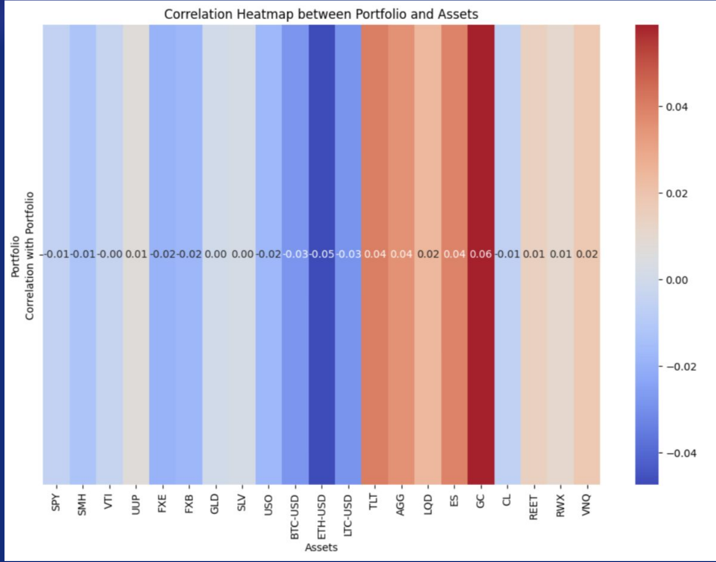


Figure 1

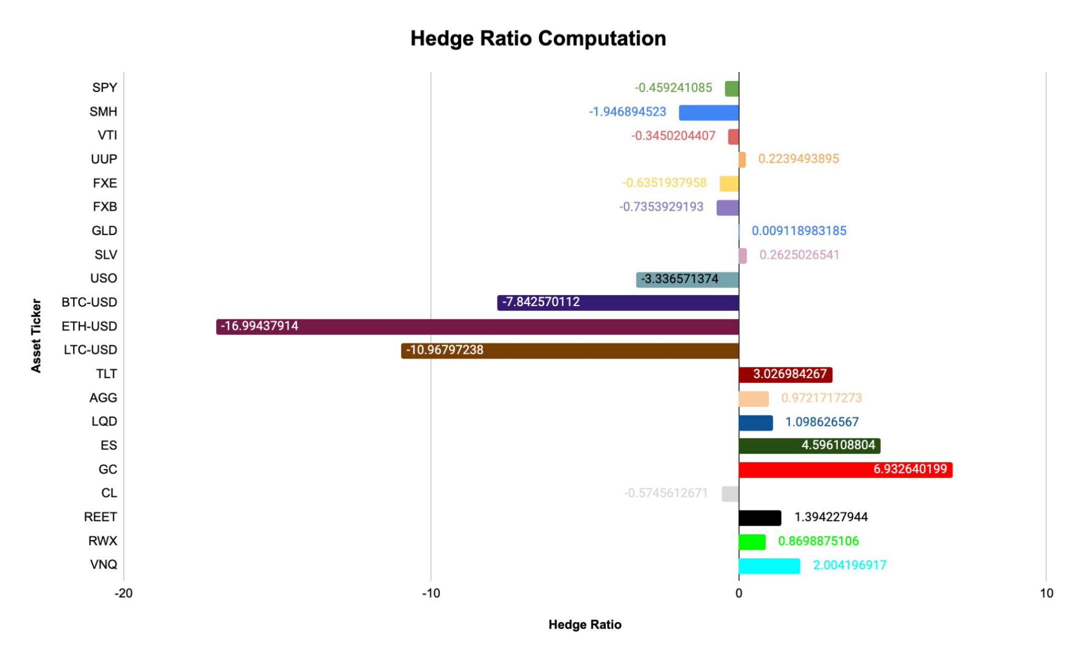
Asset-Portfolio Correlation Heatmap

The correlation heatmap visually displays the strength and direction of correlations between individual assets and the portfolio. It provides a quick overview of how closely each asset's returns move in relation to the portfolio's returns.

Figure 2

Hedge Ratio Comparison Per Asset

These bar graphs comparing the hedge ratios of all assets, providing a visual comparison of their proportional exposure within the portfolio. This allows for quick identification of assets with higher or lower hedge ratios



Results (continued)

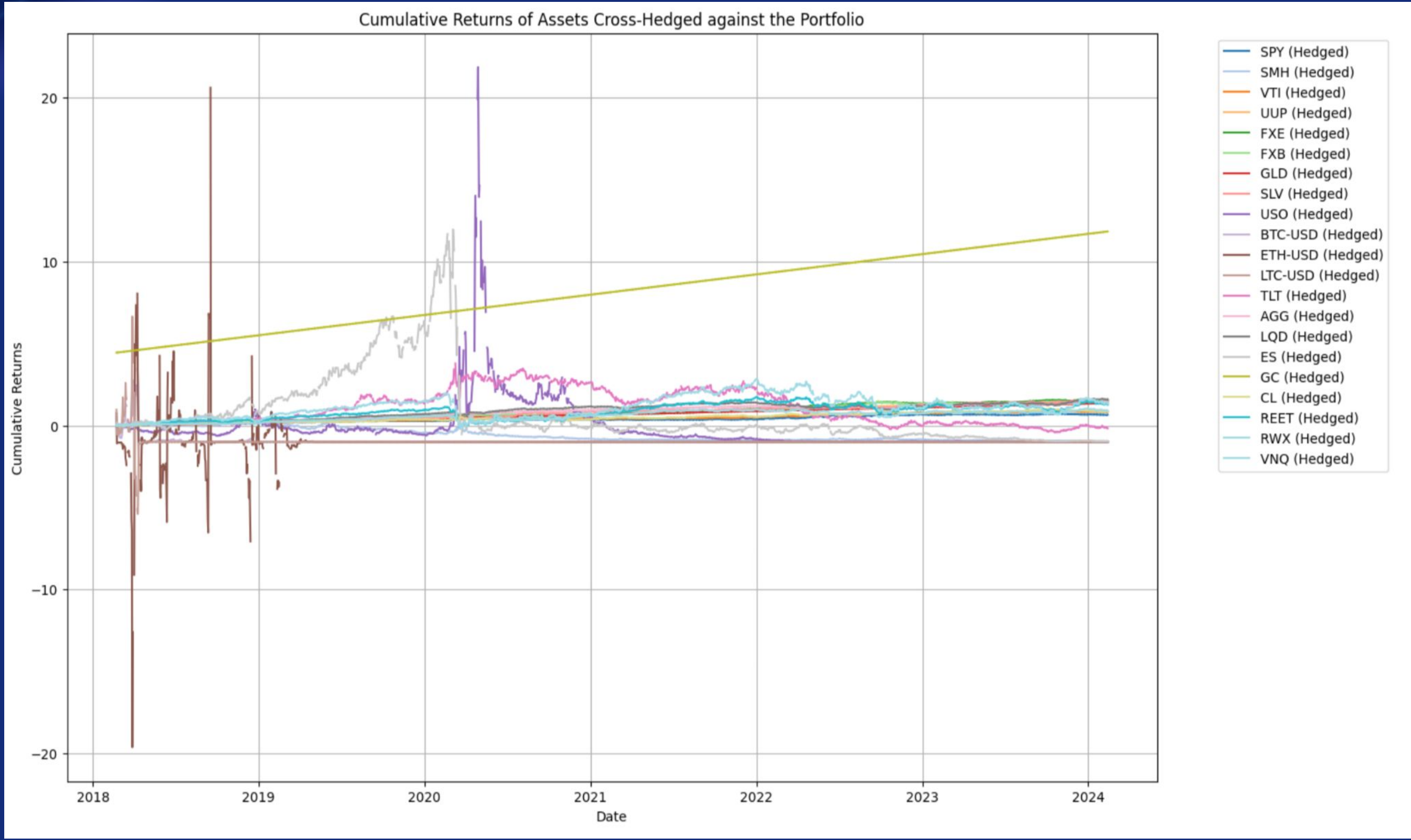


Figure 3
Cumulative Returns Timeplot for the Portfolio Hedged against each Potential Asset

Time plot illustrating the performance of the portfolio hedged against each potential asset over time. This visualization allows for a comparison of the cumulative returns achieved when employing different assets for hedging purposes. Returns can be positive if profits exceed the risk undertaken or negative which is vice versa.

Results (continued)

Figure 4
Performance Metrics Table

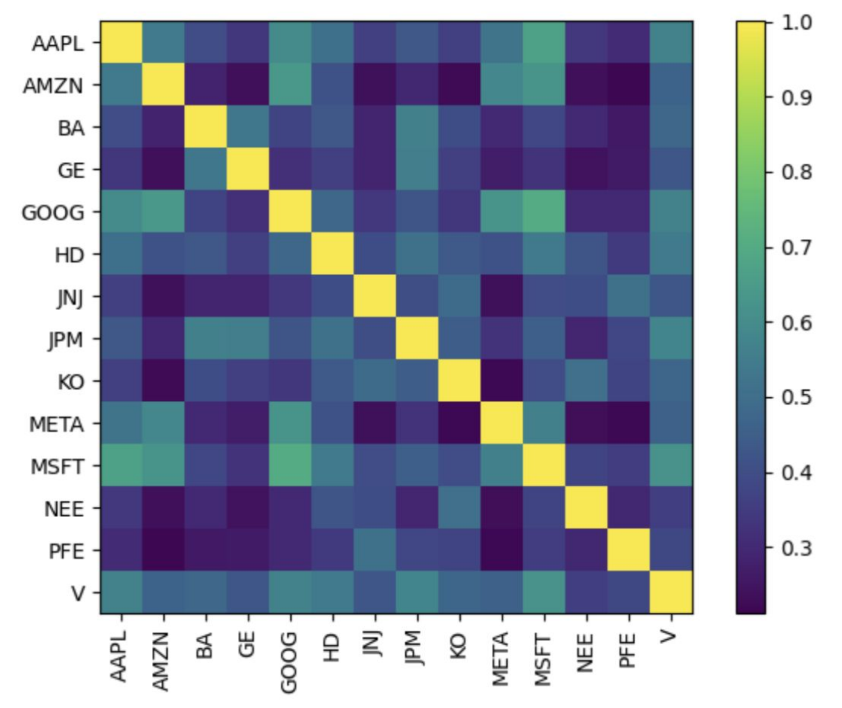
The given table presents a performance metrics table, including key indicators such as the Sharpe ratio (risk-adjusted return), annualized return (average annual return), annualized standard deviation (volatility), and maximum drawdown (peak-to-trough decline) for each asset.

	Sharpe Ratio	Annualized Return	Annualized Standard Deviation	Max Drawdown
SPY	0.9422606677	0.08639724121	0.09169144396	-0.1447432463
SMH	-0.4704308743	-0.3050478997	0.6484436214	-0.976137579
VTI	1.435036529	0.1012902354	0.07058373311	-0.09987948978
UUP	9.468233246	0.1489680713	0.01573346023	-0.0107205538
FXE	3.393729009	0.155305007	0.04576234772	-0.04914910249
FXB	2.192787823	0.148307727	0.06763432625	-0.08601100686
GLD	54.96556685	0.1392666774	0.002533707653	0
SLV	2.051377242	0.1498286182	0.07303806202	-0.1010220521
USO	0.2124154159	0.301887678	1.4212136	-0.9994039487
BTC-USD	-0.2421648429	-1.169132389	4.827837	-1.000097616
ETH-USD	0.05942627162	0.7946714793	13.37239335	-11.58518641
LTC-USD	0.4532830515	4.137894539	9.128721061	-1.573060568
TLT	0.1858267256	0.09188462021	0.4944639686	-0.8784393713
AGG	2.512675209	0.1476884638	0.05877737929	-0.09094954124
LQD	1.441991708	0.1580387917	0.1095975732	-0.2385383176
ES	0.2005798461	0.2382698224	1.187905102	-0.997207082
GC	0.4796740433	0.8581650901	1.789058846	0
CL	0.9248012788	0.107698169	0.1164554715	-0.1200932293
REET	0.5944369033	0.1793027718	0.3016346576	-0.5911787146
RWX	0.7392562493	0.1157064077	0.156517321	-0.3923841363
VNQ	0.5356227947	0.2536435294	0.4735487957	-0.7328312937

Results (continued)

Figure 5
Covariance Matrix

By accurately quantifying the relationships between assets within the portfolio, the covariance matrix aids with assets being allocated appropriately, minimizing the impact of any potential misallocation on the research outcomes. This helps maintain the integrity of the analysis and ensures that the results accurately reflect the effectiveness of the selected cross-hedging assets.



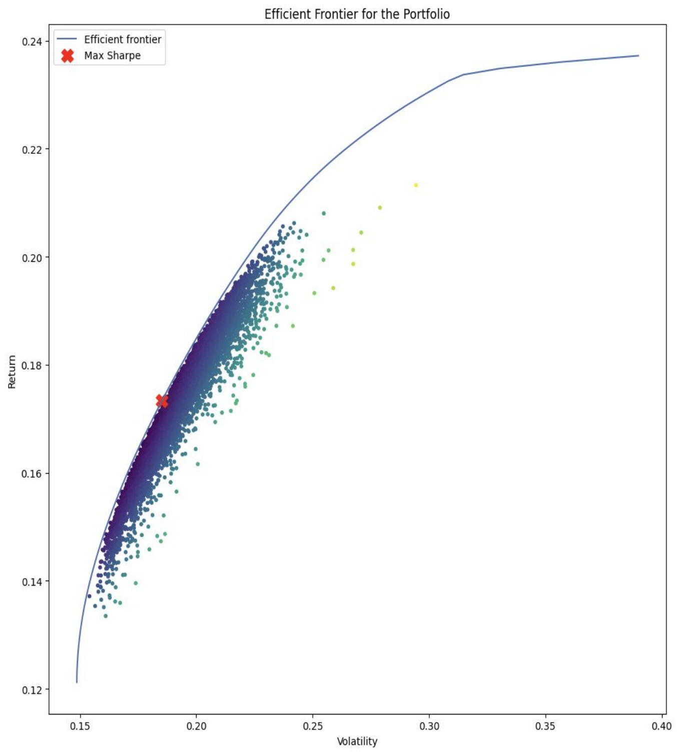


Figure 6
Efficient Frontier Curve

The efficient frontier curve illustrates the set of optimal portfolios that offer the highest expected return for a given level of risk, or the lowest risk for a desired level of return. It represents the trade-off between risk and return, showing investors the range of possible portfolio combinations to achieve their investment goals. The maximum Sharpe ratio point on the efficient frontier identifies the portfolio with the highest risk-adjusted return, providing investors with the optimal balance between risk and reward.

Results (continued)

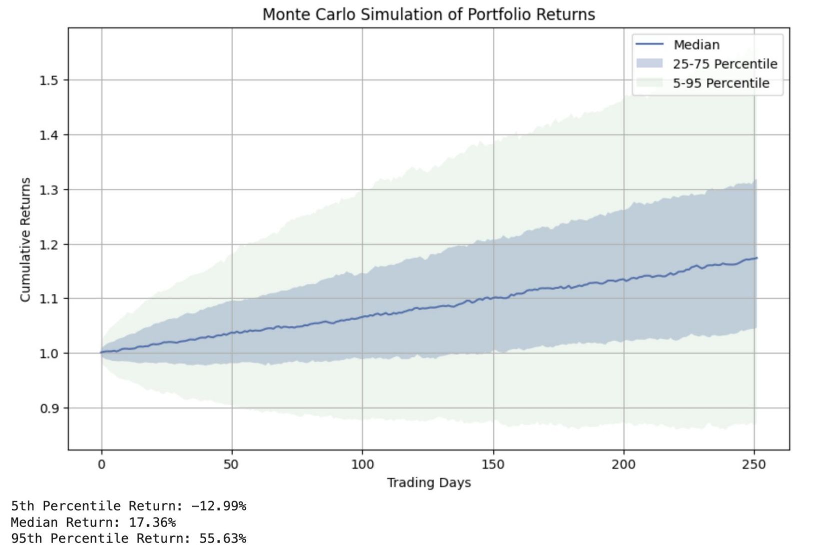


Figure 7

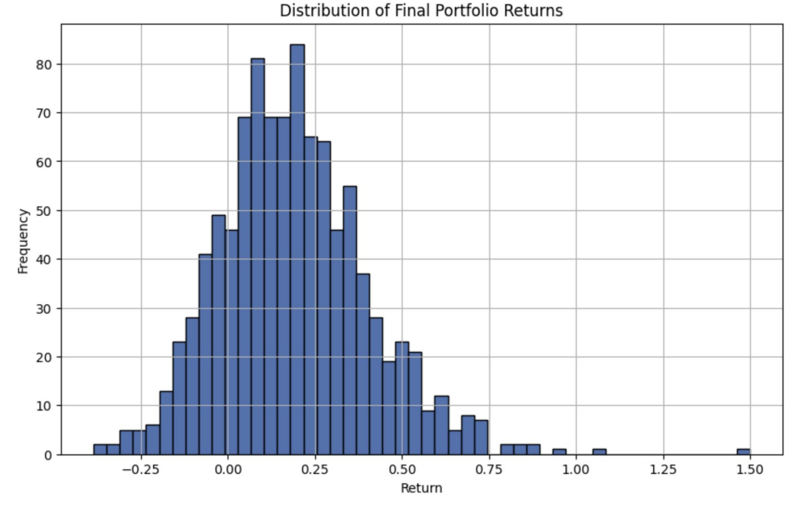
Monte Carlo Simulation of Portfolio Returns

This sector-line graph illustrates the results of a Monte Carlo simulation for portfolio returns over 250 trading days. The blue line represents the median return, while the shaded areas show the 25-75 percentile range (darker blue) and the 5-95 percentile range (lighter blue). The graph demonstrates the potential range of cumulative returns over time, with the 5th percentile return at -12.99%, the median return at 17.30%, and the 95th percentile return at 55.63%. This visualization helps investors understand the range of possible outcomes and the uncertainty associated with portfolio performance for specific cases.

Figure 8

Distribution of Final Portfolio Returns

This histogram displays the frequency distribution of final portfolio returns based on the Monte Carlo simulation. The x-axis represents the range of possible returns, while the y-axis shows the frequency of each return outcome. The distribution appears to be roughly normal but slightly right-skewed, with the majority of returns concentrated between 0% and 50%. This visualization provides insights into the probability of achieving different levels of returns, helping investors assess the risk and potential reward of the portfolio allocation strategy.



Discussion

- Given that an ideal cross-hedging asset would be negatively correlated with the portfolio at hand at a high magnitude, the heatmap suggests that Ethereum (ETH-USD) is the strongest instrument with a PPMCC value of -0.05. Following it closely are the other 2 cryptocurrency tickers (BTC-USD and LTC-USD) at -0.03 respectively. This gives us an idea that cryptocurrency could be used to offset losses as the portfolio moves in opposite directions to it. On the other hand, futures and bonds were generally observed to be poorly correlated in the context of our portfolio with high magnitude directly proportional relations.

- The hedge ratio values that were computed for each asset indicate that all cryptocurrencies are suitable for cross-hedging against the portfolio. Notably, these cryptocurrencies exhibit very high negative hedge ratios (ETH-USD: -16.99, BTC-USD: -7.84, LTC-USD: -10.97), indicating a short position with the portfolio's returns. Meanwhile, futures, bonds and real estate attested strong positive ratios, which again, underscores their weakness as hedging instruments. Traditional equities were measured to have negative yet low magnitude values as the portfolio itself was composed of a similar asset class. On a different tangent, the variation in currency index ratios could be explained by difference in their dependence and exposure to global economic factors and geopolitical events.

- Initial fluctuations in the cumulative returns plot are eventually damped, creating a split of evenly divided positive and negative cumulatives. Notably, these fluctuation coincided with the rise of the COVID-19 pandemic which would have meant that the portfolio experienced significant volatility during this period, reflecting the broader market turmoil caused by the pandemic. Despite this initial turbulence, the cumulative returns stabilized over time, showcasing the resilience of the portfolio in the face of adversity.

Conclusion: Implications

- While the assessment of specific performance metrics for cryptocurrencies, notably the Sharpe Ratio and Annualized Return, may suggest weak performance within the given timeframe, it's pertinent to contextualize these findings. Firstly, the negative values, albeit marginal (if they exist in the first place), imply that the risk may slightly outweigh the return, yet not significantly so. Secondly, attributing this performance solely to the COVID-19 pandemic underscores its exceptional and unprecedented nature, which could have distorted market dynamics and led to atypical outcomes. Therefore, it's essential to interpret these results cautiously, considering the extraordinary circumstances surrounding the evaluation period.

- Thus, even though it can be posited that cryptocurrencies could be ranked as the most reliable asset class in the context of cross-hedging, it would be premature to conclusively assert that they are best type based solely on these observations. It is constrained by many factors which limit the applicability and generalization of the relevance of the conclusions to past or future events.

- Quite similarly, commodities such as silver, gold and oil can be classed as dependable assets for cross-hedging due to their historical performance and inherent value as tangible assets, which are also fundamental necessities in various industries and sectors.

Conclusion: Limitations

- **Single-Country Focus:**
  - ➔ Our analysis focuses on one country's assets, potentially overlooking cross-hedging nuances with international assets.
  - ➔ Future research could explore cross-hedging efficacy with international assets, considering regulatory and market differences.
- **Sectoral Focus:**
  - ➔ Our study mainly examines cross-hedging within sectors of one country.
  - ➔ Diversifying across sectors may offer more risk mitigation opportunities, warranting further investigation.
- **Portfolio Diversity Quantification:**
  - ➔ While our study covers diverse assets, quantifying portfolio diversity could provide deeper insights.
- **Regulatory Environment Impact:**
  - ➔ Less regulated markets' inclusion introduces risks not fully captured in our analysis.
  - ➔ Following research could assess regulatory environments' impact on cross-hedging effectiveness.
- **Rare Events Consideration:**
  - ➔ Rare events like the COVID-19 pandemic, though infrequent, can significantly affect markets and analysis outcomes.
- **Data Timeframe Sensitivity:**
  - ➔ Data timeframe impacts analysis relevance and reliability as it make the results applicable better to those time periods.
  - ➔ Next steps could be to explore data sensitivity to different periods and market conditions.
- **Model Hindrances:**
  - ➔ Model complexities may introduce biases or limitations.
  - ➔ Higher-quality data may enhance accuracy, but it's essential to scrutinize model parameters that could interfere with results.

Conclusion: Next Steps

1. Probability Distribution Analysis:
   - Utilizing probability distribution curves to model and quantify risk associated with each potential asset type. This approach enables deeper insights into volatility characteristics and their impact on portfolio performance.

2. Exploration of Alternative Risk Management Strategies:
   - Investigating alternative risk management strategies, including pairs trading, to identify optimal asset types. Comparing the effectiveness of various strategies under different market conditions to determine the most suitable assets for risk mitigation within our portfolio.

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